A SURVEY OF BALLISTIC TRANSFERS TO LOW LUNAR ORBIT

Jeffrey S. Parker[*]

Rodney L. Anderson[*] **and Andrew Peterson**[†]

A simple strategy is identified to generate ballistic transfers between the Earth and Moon, i.e., transfers that perform two maneuvers: a trans-lunar injection maneuver to depart the Earth and a Lunar Orbit Insertion maneuver to insert into orbit at the Moon. This strategy is used to survey the performance of numerous transfers between varying Earth parking orbits and varying low lunar target orbits. The transfers surveyed include short 3-6 day direct transfers, longer 3-4 month low-energy transfers, and variants that include Earth phasing orbits and/or lunar flybys.

INTRODUCTION

There are many ways to transfer a spacecraft from the Earth to an orbit about the Moon. The conventional, direct method has been implemented in many missions, including the Surveyor and Apollo missions as well as the more recent Lunar Reconnaissance Orbiter mission, among others. The method involves a large maneuver to depart the Earth, a large maneuver to insert into orbit at the Moon, and up to six days of transfer duration. The spacecraft performs small trajectory correction maneuvers, but otherwise coasts ballistically during the transfer. The GRAIL mission[1,2,3] is implementing a low-energy transfer[4] that is operationally similar to the direct transfer; that is, it involves a large maneuver to depart the Earth, a large maneuver to insert into orbit at the Moon, and a ballistic coast. The transfer requires less fuel, given the same propulsion system, though it requires 2–3 additional months of transfer duration.

A variety of trajectory types follow this paradigm. The direct and low-energy trajectories may include any number of Earth phasing orbits and/or lunar flybys prior to their final lunar arrival. Each phasing orbit in a lunar transfer increases the duration of the transfer and potentially adds undesirable passes through the Van Allen Belts, but also opens up new options in the mission design trade space. Earth phasing orbits often permit a mission to launch on multiple days to reach a particular lunar arrival, particularly if orbit-raising maneuvers are introduced in the mission plan, e.g., ISRO's Chandrayaan-1 lunar mission.[5]

At first glance, it may not be obvious when a mission can depart the Earth and/or arrive at the Moon using any given trajectory type. For instance, it may not be obvious when a mission can launch from the Earth to reach a particular lunar orbit using a 3-month long low-energy transfer via the Sun-Earth L_1 point. Furthermore, it may not be obvious how much ΔV is required, and whether or not it is worth the added transfer duration to follow the low-energy transfer rather than a direct lunar transfer. The purpose of this paper is to provide a survey of lunar transfers that follow the two-burn ballistic transfer paradigm (i.e., transfers that are entirely ballistic except for the Earth-departure and lunar-arrival maneuvers). This paper will consider only missions to polar lunar orbits,

---
[*]Member of Technical Staff, Jet Propulsion Laboratory, California Institute of Technology, M/S 301-121, 4800 Oak Grove Dr., Pasadena, CA 91109

[†]Research Assistant, Georgia Institute of Technology, Atlanta, GA 30332

in order to reduce the scope of the study. The surveys presented here illustrate when and how to depart the Earth in order to arrive at a given polar lunar orbit at a given time. They also illustrate the range of fuel costs associated with each transfer type as well as the required itinerary of Earth phasing orbits and/or lunar flybys (if any). This information will guide mission designers as they select a particular lunar transfer – or symmetric Earth return – for a mission.

Background

Some of the most famous two-burn nearly-ballistic lunar transfers were those used in the Apollo program. Figure 1 illustrates the 3-day transfer that the Apollo 11 astronauts used to go from the Earth to the Moon in 1969.[6] The mission implemented a low Earth parking orbit with an inclination of approximately 31.38°. From there, the launch vehicle was required to attain a trans-lunar injection C_3 of approximately -1.38 km^2/s^2 to reach the Moon in approximately 3.05 days. Upon arrival at the Moon, the vehicle injected into an elliptical orbit with a periapse altitude around 110 km and an apoapse altitude around 310 km, followed soon after by a circularization maneuver.[6] In order to compare the Apollo 11 transfer with the transfers in the surveys presented here, the Apollo 11 transfer would have a velocity of approximately 2.57 km/s at an altitude of 100 km above the mean lunar surface, requiring a hypothetical, impulsive ΔV of approximately 0.94 km/s to insert into a circular 100-km orbit.
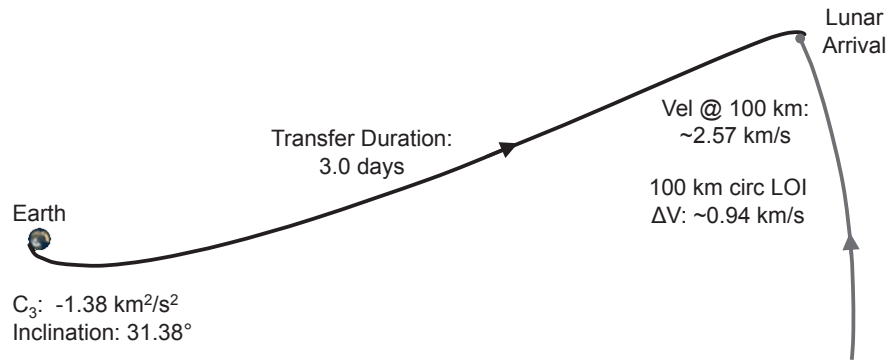

Figure 1 **The Apollo 11 Earth-Moon transfer, as if it had performed an impulsive LOI maneuver directly into a circular, 100-km lunar orbit.**

The GRAIL mission design provides an alternative two-burn nearly-ballistic lunar transfer design.[1] The two GRAIL spacecraft are injected into nearly identical Earth-departure orbits from a low Earth parking orbit, much like conventional, direct lunar transfers. However, they are injected at higher velocities such that their initial orbits have apogees far beyond the orbital radius of the Moon. As each spacecraft traverses its orbit's apogee, the Sun's gravity pulls on it and raises its orbit's perigee altitude. Both spacecraft pass near the Sun-Earth L_1 point, EL_1. By the time each spacecraft falls back toward the Earth, its perigee altitude has risen high enough such that it encounters the Moon and can immediately perform a lunar orbit insertion. GRAIL's launch vehicle targets a trans-lunar injection C_3 of -0.65 ± 0.1 km^2/s^2 to obtain a cruise that arrives at the Moon between 90 and 115 days later, where both parameters vary depending on the launch date. Both GRAIL spacecraft arrive at the Moon with very similar velocities across the launch period; each spacecraft performs an initial orbit insertion to become captured by the Moon. From there they perform a series of maneuvers to transition into their science orbit.[3] In order to compare the GRAIL trajectory with

those surveyed in this paper, and with the Apollo transfer discussed above, each GRAIL spacecraft would have a velocity of approximately 2.30 km/s at an altitude of 100 km if they did not initiate their orbit insertion before that point. A hypothetical impulsive orbit insertion maneuver would require a ΔV of approximately 0.67 km/s to insert each spacecraft into similar circular 100-km lunar orbits.

METHODOLOGY

Each transfer in the surveys presented in this paper departs the Earth, coasts to the Moon, and injects directly into a low lunar orbit. To reduce the scope of the problem while still yielding practical data, the surveys presented here assume that the mission targets a circular 100-km polar orbit about the Moon. This lunar orbit is akin to the mapping orbits of several spacecraft, including Lunar Prospector, Kaguya/SELENE, Chang'e 1, Chandrayaan-1, and the Lunar Reconnaissance Orbiter.

The lunar orbit insertion (LOI) is modeled as a single impulsive maneuver that is performed at the periapse point and places the vehicle directly into a circular orbit. This is not a realistic maneuver, but it is useful to directly compare the total insertion cost of one transfer to another. The orbit insertion cost needed to place a satellite into an elliptical orbit, rather than a circular orbit, may be determined via the Vis-Viva equation.[7]

The surveys presented in this paper have been generated using a method that does not make many assumptions about what the lunar transfers look like. This permits each survey to reveal trajectories that may not have been expected. Each trajectory in each survey is constructed using the following procedure:

1. Construct the target lunar orbit. The following parameters are used in this study, specified in an inertial coordinate system that is centered at the Moon and aligned with the mean lunar spin axis, according to the standards defined by the International Astronomical Union (IAU):

$$
\begin{aligned}
\text{Periapse radius, } r_p &: \quad 1837.4 \text{ km } (\sim100 \text{ km altitude}) \\
\text{Eccentricity, } e &: \quad \epsilon \\
\text{Equatorial inclination, } i &: \quad 90° \\
\text{Argument of periapse, } \omega &: \quad \text{Specified value} \\
\text{Longitude of the ascending node, } \Omega &: \quad \text{Specified value} \\
\text{True anomaly, } \nu &: \quad 0°
\end{aligned}
$$

   The argument of periapse is undefined for a circular orbit. However, since all practical missions to date have inserted into an elliptical orbit, and some missions remain in a highly elliptical orbit, the target orbit's argument of periapse, ω, is presented here rather than the true anomaly, which is kept at $0°$ to indicate that LOI is performed at periapse. The orbit's eccentricity is given as ϵ to indicate that it is nearly circular, while permitting ω to be defined.

2. Construct the LOI state.

   (a) Specify the date of the LOI, t_{LOI}. Dates are given here in ET (Ephemeris Time).

   (b) Specify the magnitude of the impulsive orbit insertion maneuver, ΔV_{LOI}. Apply the ΔV in a tangential fashion to the LOI state.

3. Propagate the state backward in time for 200 days.

4. Identify the perigee and perilune passages that exist in the trajectory.

   (a) If the trajectory flies by the Moon within 500 km, label the trajectory as undesirable.

   (b) The latest perigee passage that approaches within 500 km of the Earth is considered the earliest opportunity to inject into that trajectory.

   (c) If no low perigees are observed, then the lowest perigee is identified as the trans-lunar injection (TLI) location.

5. Characterize the performance of the trajectory, making note of the following values:

   - TLI altitude, inclination, and C_3;
   - Duration of the transfer;
   - Periapse altitude of any/all Earth and Moon flybys; and
   - LOI ΔV magnitude.

This procedure requires four inputs: the longitude of the ascending node of the target orbit (Ω), the argument of periapse of the target orbit (ω), the ΔV of the impulsive LOI (ΔV_{LOI}), and the date of the LOI (t_{LOI}). Figures 2 and 3 show two examples of lunar transfers generated with this procedure. Figure 2 illustrates a direct 4-day transfer and Figure 3 illustrates an 84-day low-energy transfer. The inputs and performance parameters of these transfers are summarized in Table 1.
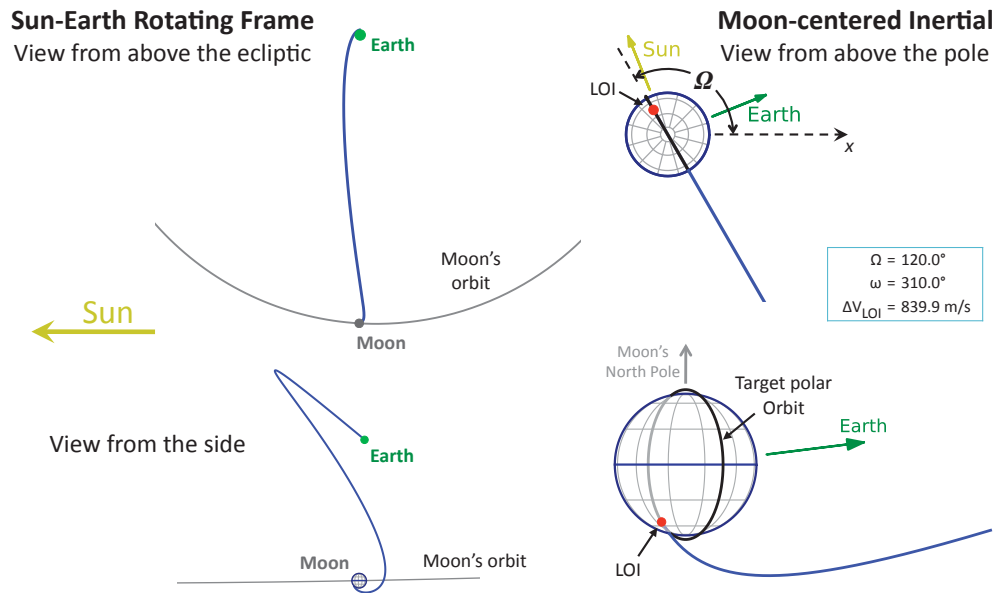

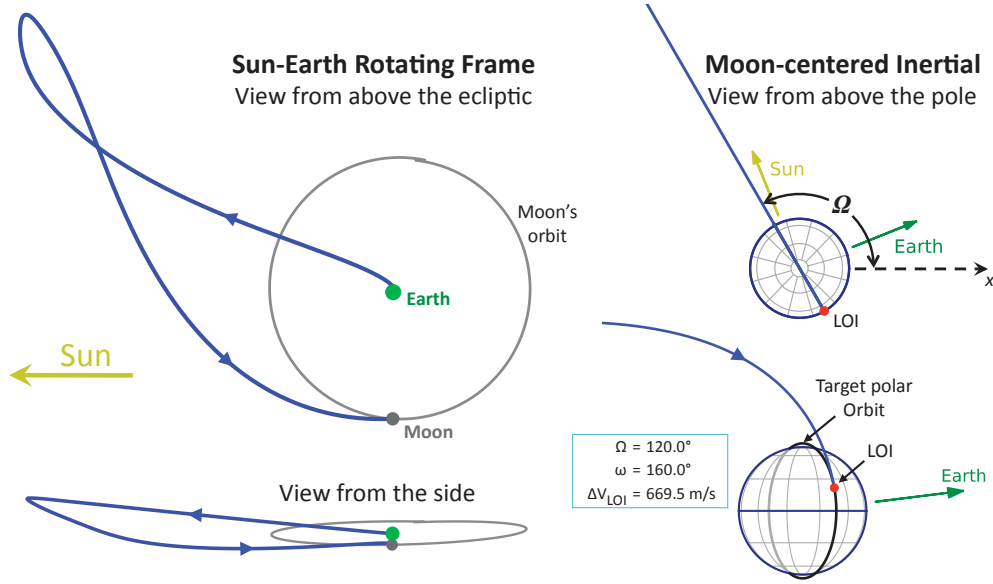
Figure 2. An example 4-day direct lunar transfer.

Figure 3. An example 84-day low-energy lunar transfer.

Many surveys have been conducted, searching for practical lunar transfers. In general, a survey fixes the parameters Ω and t_{LOI} and systematically varies the other two parameters. This process generates a two-dimensional map displaying a parameter – typically the TLI altitude – which changes smoothly as either Ω or t_{LOI} shift. These surveys will be described in more detail in the next sections.

Table 1 The inputs and performance parameters of the two example lunar transfers shown in Figures 2 and 3. Both transfers begin in a 185 km circular LEO parking orbit before their injections, and both transfers arrive at the Moon on July 18, 2010 at 9:50:08 ET.

Figure #	Ω (deg)	ω (deg)	ΔV_{LOI} (m/s)	Duration (days)	LEO Inclination (deg) Equatorial	Ecliptic	C_3 (km^2/s^2)
2	120.0	310.0	839.878	4.036	62.114	39.761	-2.064
3	120.0	160.0	669.543	83.706	28.093	5.921	-0.725

EXAMPLE SURVEY

Figure 4 shows the results of an example survey of lunar transfers. In this example, Ω is set to $120°$, the LOI date is set to 18 July 2010 09:50:08 ET, the value of ω is systematically varied from $0° - 360°$, and ΔV_{LOI} is systematically varied from $650 - 1050$ m/s, a range empirically determined to generate practical transfers. Figure 4 shows the altitude of the trans-lunar injection point for each combination of ω and ΔV_{LOI}. The points colored in white correspond to trajectories that arrive at the Moon such that when propagated backward in time they never come any closer to the Earth. The

points colored in black correspond to trajectories that arrive at the Moon such that when propagated backward in time they approach within 10,000 km of the Earth: trajectories that may be used to generate real missions,[8] assuming the departure time and geometry are acceptable.
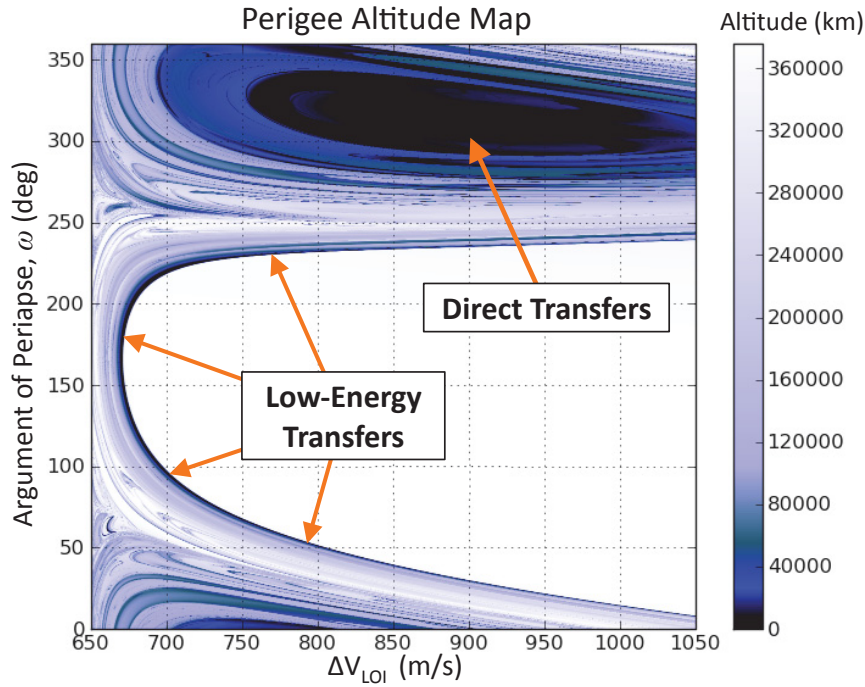

Figure 4   The altitude of the TLI location for each combination of ν and ΔV_{LOI}, given a lunar orbit insertion on July 18, 2010 into a lunar orbit with Ω equal to $120°$.

The plot shown in Figure 4 contains many interesting features. First of all, roughly half of the state space is white, corresponding to trajectories that arrive at the Moon from heliocentric orbits. With a quick investigation, one finds that the large black field toward the top of the plot corresponds to direct transfers to the Moon, i.e., trajectories that take 2-6 days to reach the Moon, much like Apollo and LRO; though most of the trajectories include Earth phasing orbits that extend the transfer's duration. The black curve that outlines the large white field corresponds to low-energy lunar transfers that require 80-120 days. There are many other curves throughout the plot that correspond to trajectories that enter some sort of large Earth orbit, or perform a combination of one or more flybys.

The direct transfers that are observed in the upper part of the plot shown in Figure 4 require ΔV_{LOI} values from 760 m/s to 1000 m/s or more. The direct transfers that don't involve any Earth phasing orbits or any sort of lunar flyby require at least 818 m/s, though nearly all require 845 m/s or more. Figure 5 presents two similar maps, where the transfers shown in Figure 4 that approach within 1000 km of the Earth are color-coded according to the number of Earth perigee passages (top) and lunar flybys (bottom) that they make before arriving at their target orbit. One notices that direct transfers with more phasing orbits and/or lunar flybys may require less orbit insertion ΔV than the most basic lunar transfers. In any case, low-energy trajectories exist that require as little as 669 m/s, nearly 100 m/s less than the most efficient direct transfers observed and $\sim$170 m/s less than most simple direct transfers.
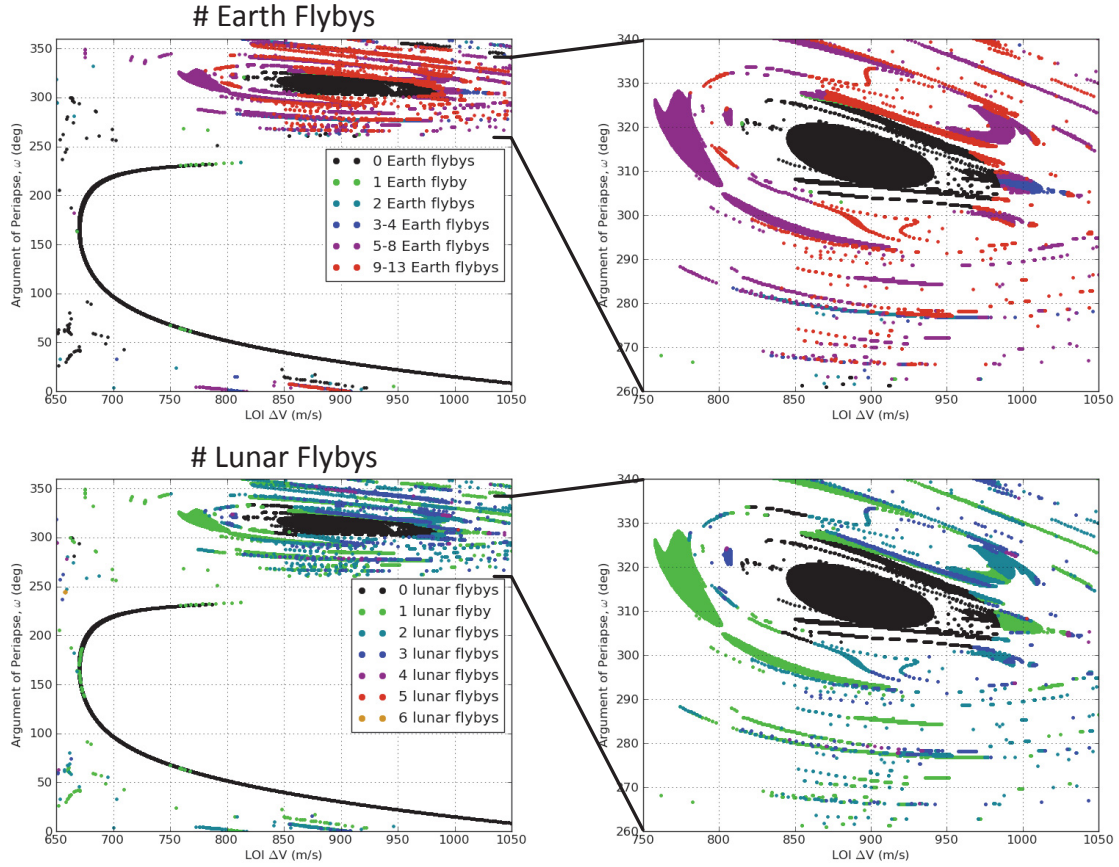
Figure 5   The transfers shown in Figure 4 that approach within 1000 km of the Earth, color-coded according to the number of Earth phasing orbits (top) and lunar flybys (bottom) that they make. The upper portion of each map, corresponding with direct lunar transfers, is magnified in the plots on the right.

Tables 2 and 3 summarize the performance parameters of several example direct lunar transfers and low-energy lunar transfers, respectively. Several examples of these trajectories are shown in Figures 6 and 7, respectively. One can see that the value of ΔV_{LOI} is over 100 m/s lower for low-energy transfers in nearly all examples, though the TLI injection energy, C_3, is higher. The injection energy of direct lunar transfers is very close to -2.0 km^2/s^2, compared to a value of approximately -0.7 km^2/s^2 for low-energy transfers. Both types of transfers have wildly varying TLI inclination values, both relative to the Earth's equator and to the ecliptic. This suggests that transfers can begin from any inclination about the Earth. Previous work has shown that one can use 1-3 maneuvers in order to adjust a trajectory to depart from a specified TLI inclination rather than the ballistic inclination value shown in the tables.[8] The total ΔV required to make this adjustment is on the order of 1 m/s per degree of inclination change.

Table 2 A summary of the performance parameters of several direct lunar transfers shown in Figure 4 and illustrated in Figure 6.

| Traj #	Ω (deg)	ω (deg)	ΔV_{LOI} (m/s)	Duration (days)	LEO Inclination (deg) Equatorial	Ecliptic	C_3 (km^2/s^2)	# Earth Flybys	# Moon Flybys
D1	120.0	321.3	818.0	4.111	22.147	8.551	-2.078	0	0
D2	120.0	326.4	860.4	4.155	43.459	62.667	-2.058	0	0
D3	120.0	304.8	867.5	4.004	85.516	63.963	-2.045	0	0
D4	120.0	301.5	947.7	3.942	142.173	123.280	-2.006	0	0
D5	120.0	311.7	971.8	4.009	131.320	154.340	-2.002	0	0
D6	120.0	321.0	813.3	13.941	24.717	6.435	-2.095	1	0
D7	120.0	326.4	868.0	14.005	52.683	72.504	-2.071	1	0
D8	120.0	279.0	870.0	32.759	19.407	31.944	-2.046	2	1
D9	120.0	325.5	758.0	67.175	37.135	13.784	-2.292	6	1
D10	120.0	327.0	810.1	84.747	62.694	39.723	-2.055	7	3
D11	120.0	354.9	828.8	85.441	75.489	54.465	-2.061	7	1
D12	120.0	268.2	861.4	141.341	46.894	63.333	-2.054	8	1
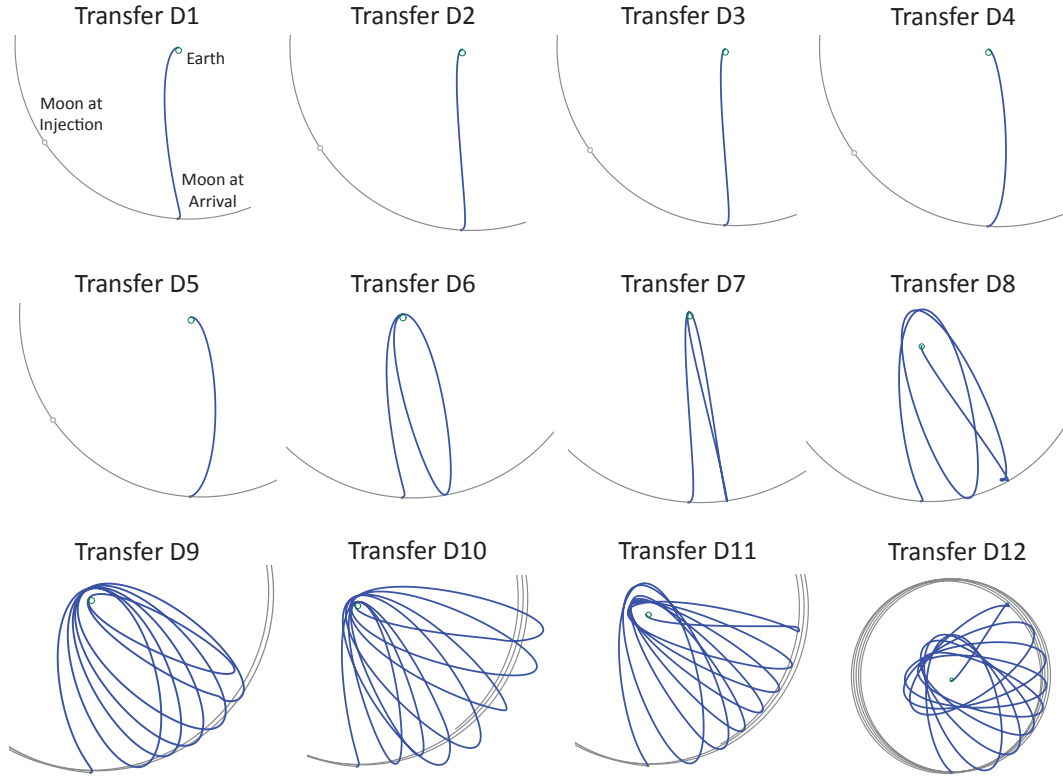
Figure 6 Example plots of several of the transfers summarized in Table 2. The trajectories are shown in the Sun-Earth rotating frame, such that the Sun is fixed on the x-axis toward the left.

Table 3   A summary of the performance parameters of several low-energy lunar transfers shown in Figure 4 and illustrated in Figure 7.

Traj #	Ω (deg)	ω (deg)	ΔV_{LOI} (m/s)	Duration (days)	LEO Inclination (deg) Equatorial	Ecliptic	C_3 (km^2/s^2)	# Earth Flybys	# Moon Flybys
L1	120.0	169.2	669.3	83.483	29.441	6.129	-0.723	0	0
L2	120.0	103.8	692.1	85.287	25.688	34.778	-0.723	0	0
L3	120.0	70.2	743.9	93.598	57.654	74.955	-0.667	0	0
L4	120.0	225.3	716.0	93.621	134.322	112.840	-0.657	0	0
L5	120.0	99.9	697.5	110.060	83.127	61.624	-0.697	0	0
L6	120.0	186.9	673.2	122.715	23.941	3.088	-0.712	0	0
L7	120.0	61.5	660.4	143.360	18.624	35.412	-0.572	0	1
L8	120.0	59.7	651.3	129.422	73.143	96.544	-0.612	0	3
L9	120.0	36.3	661.5	144.417	146.592	138.491	-0.658	0	1
L10	120.0	348.6	675.1	155.107	36.598	16.583	-0.645	5	1
L11	120.0	262.2	656.1	141.982	153.641	176.867	-0.608	0	3
L12	120.0	244.2	657.8	136.687	179.084	156.890	-0.640	0	6
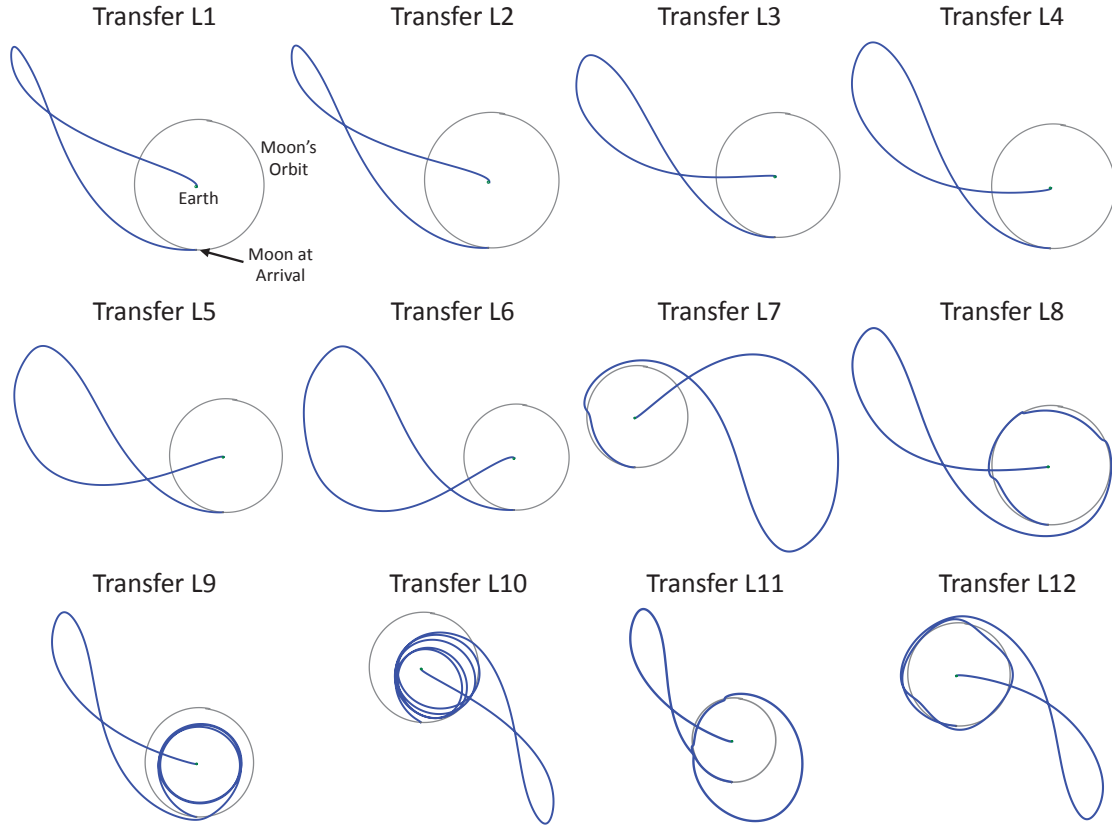
Figure 7   Example plots of several of the transfers summarized in Table 3. The trajectories are shown in the Sun-Earth rotating frame, such that the Sun is fixed on the x-axis toward the left.

ARRIVING AT A FIRST-QUARTER MOON

All of the transfers presented in the previous section arrive at the Moon at a particular time into a particular orbit, namely, a circular, polar orbit with a longitude of the ascending node, Ω, of 120° and a time of arrival, t_{LOI}, of July 18, 2010 at 9:50:08 ET. This time of arrival corresponds to a moment in time when the Sun-Earth-Moon angle is equal to approximately 90° at the Moon's 1st quarter. This is very similar to the arrival geometry of the two GRAIL spacecraft, though it is a different month. In addition, the plane of the target orbit is nearly orthogonal to the Earth-Moon line. A polar orbit with an Ω-value of 111.9° (also 291.9°) is in a plane that is as close to orthogonal to the Earth-Moon line as a polar orbit can get on this date. The surveys presented in this section keep the time of arrival the same and explore the changes to the lunar transfers that occur as the target orbit's Ω-value is varied.
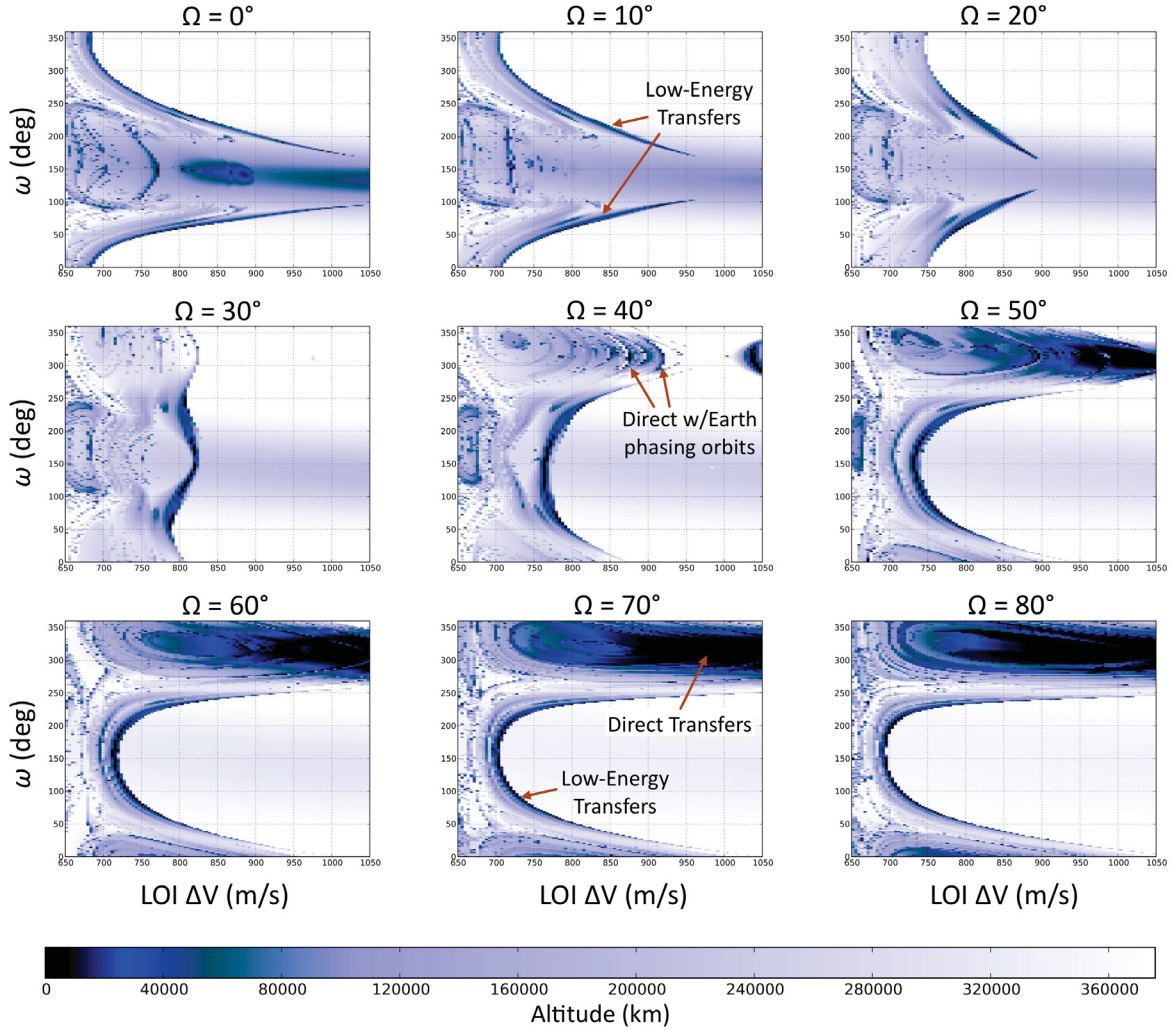

Figure 8 Nine surveys of trajectories that arrive at the first-quarter Moon, where the target orbit's Ω varies from $0° - 80°$. Points in black originate from the Earth; other points are colored according to how close they come to the Earth when propagated backward, using the color scheme presented in Figure 4.

Figures 8 and 9 show surveys of the lunar transfer state space as Ω varies from $0° – 80°$ and $160° – 270°$, respectively. There is a clear progression of the state space as Ω varies. Locations where direct and low-energy transfers exist are indicated. The state space varies much less discernibly when Ω is within $\sim30°$ of $111.9°$ or $291.9°$, namely, when the orbit is close to being orthogonal to the Earth-Moon line.

Many features are quickly apparent when studying the maps shown in Figures 8 and 9. First, a large portion of each map is white, corresponding to combinations of ΔV_{LOI} and ω that result in trajectories that depart the Moon backward in time and traverse away from the Earth-Moon system. At lower ΔV_{LOI}-values, the trajectories depart the Moon backward in time and later impact the Moon or remain very near the Moon. One can see curves of black in each map, corresponding to trajectories that depart the Moon backward in time and eventually come very near the Earth; hence, making viable Earth-Moon transfers. The features are observed to shift in a continuous fashion across the range of Ω-values.

If one surveys these maps, one finds that low-energy transfers exist to any lunar orbit plane, but simple direct transfers only exist for certain ranges of Ω-values. Direct transfers can only reach orbits with Ω-values between approximately $50°$ and $170°$ and between approximately $230°$ and $350°$. These orbit planes are within about $60°$ of being orthogonal to the Earth-Moon line; furthermore, direct lunar transfers require less ΔV for their orbit insertions the closer they are to being orthogonal to the Earth-Moon line.

Figure 10 captures the least-expensive ΔV_{LOI} for direct lunar transfers as well as low-energy lunar transfers for any target orbit plane studied. There are many trajectories that require more ΔV, but this figure tracks the least expensive transfer in each case. Trajectories with Earth phasing orbits and/or lunar flybys may require even less ΔV, but those are not tracked here since there are so many paths that a spacecraft can take through the system. One observes that low-energy transfers do indeed reach any target orbit, though the insertion ΔV costs vary as the orbit plane changes. Direct lunar transfers are indeed limited to certain orbital planes, and they require at least 140 m/s more LOI ΔV than a low-energy transfer to the same orbit. The transfer durations for the simple, direct transfers shown here require between 2 and 12 days; the low-energy transfers require between 70 and 120 days.

The lunar transfers with the least LOI ΔV and no low Earth or lunar periapse passages have been identified for each combination of Ω and ω; their performance parameters are plotted in Figure 11. The left plot shows a map of the LOI ΔV cost of these transfers; the plot on the right shows the corresponding transfer duration for each trajectory. The low-ΔV solutions identified in Figure 10 are plotted in these maps for reference, and to identify their ω-values and durations. Direct transfers are easily discerned by observing the dark fields in the plot on the right, corresponding to short-duration transfers. One can see that there are large fields of combinations of Ω and ω that yield low-energy transfers, though the costs increase as one moves away from the low-ΔV curves. One can see that the combinations of Ω and ω that yield practical direct transfers are much more limited.

The maps shown in Figure 11 are very useful: they illustrate what sorts of transfers may be used to reach any given polar orbit at the Moon, given that the transfers must arrive at the Moon at this particular arrival time. Missions that target an elliptical orbit must consider which argument of periapsis value to target; missions which aim to enter a circular orbit may likely use any ω for the initial orbit insertion, simplifying the trade space. Similar maps may be generated for any lunar arrival time: two different arrival times will be considered in the next sections.
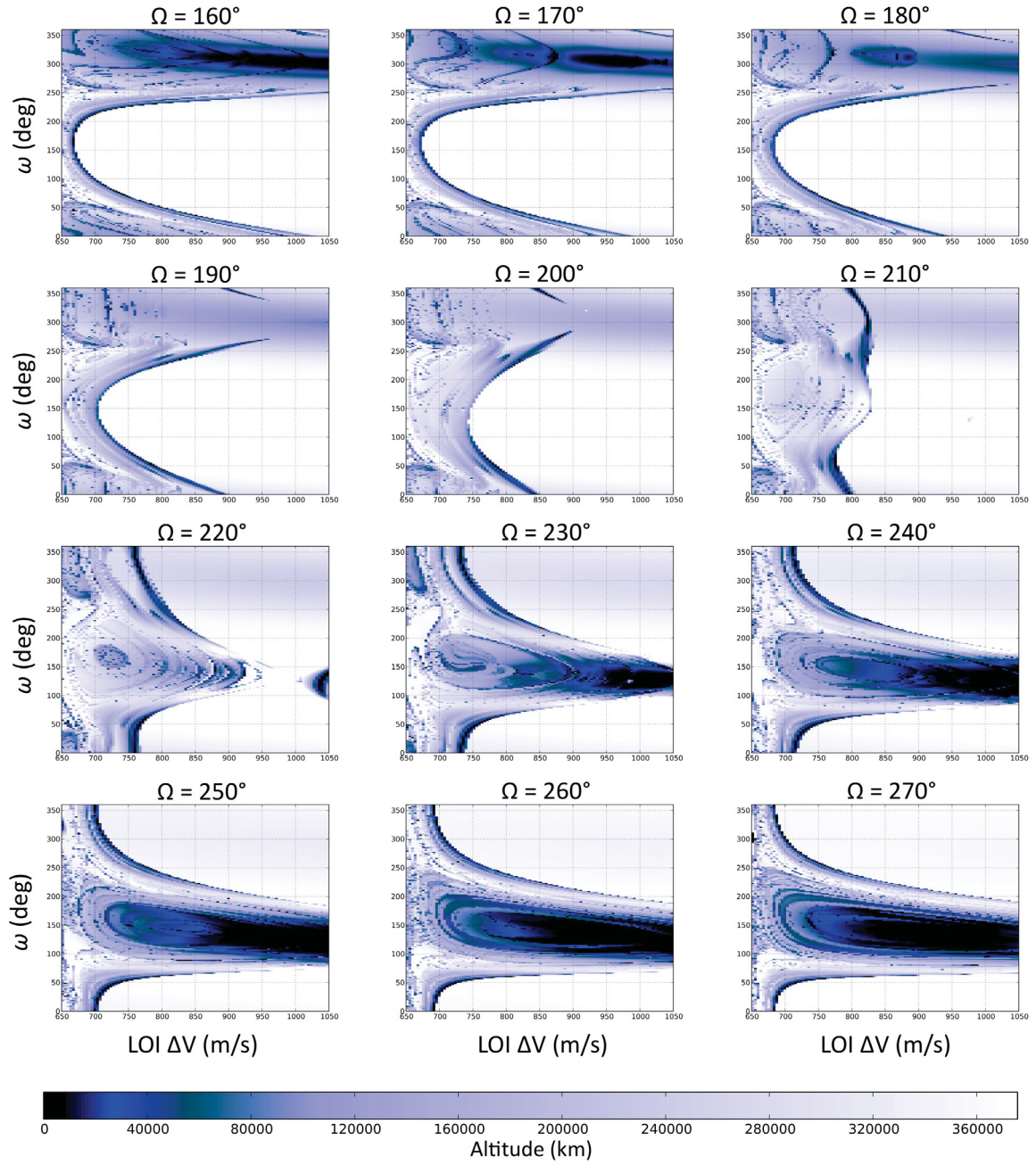
Figure 9 Twelve surveys of missions that arrive at the first-quarter Moon, where the target orbit's Ω varies from 160° – 270°. The maps are colored according to the closest approach distance that the trajectories make with the Earth, as illustrated in Figures 4 and 8.
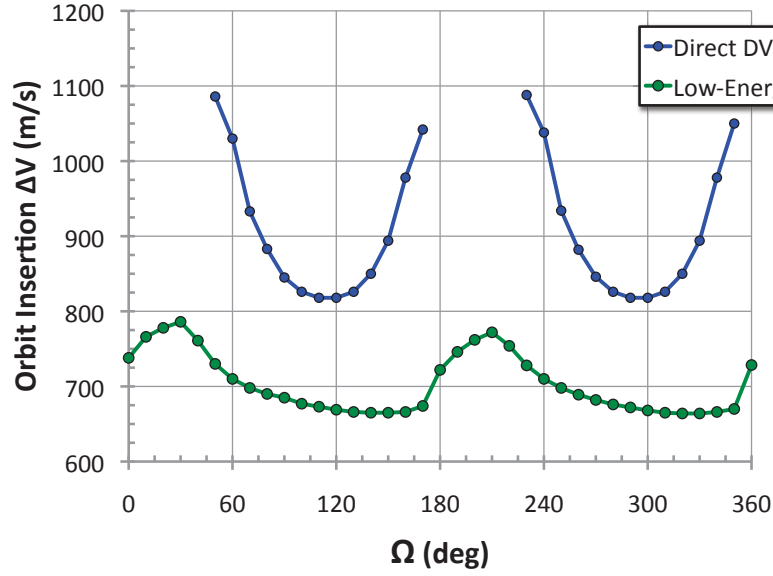
Figure 10   The minimum lunar orbit insertion ΔV for direct and low-energy lunar transfers, requiring no Earth phasing orbits nor lunar flybys for transfers to a first-quarter Moon.
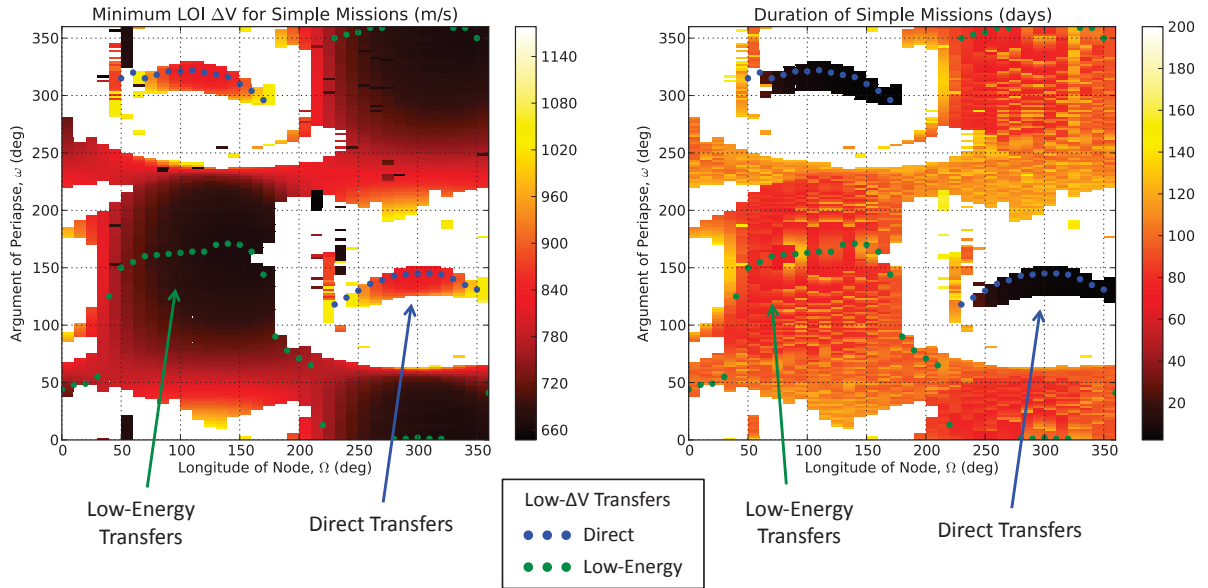

Figure 11   The combinations of Ω and ω that yield simple lunar transfers, i.e., those without low Earth or lunar periapse passages. If multiple transfers exist for the same combination, then the one with the least LOI ΔV is shown. All of these transfers arrive at a first-quarter Moon. The low-ΔV transfers shown in Figure 10 are indicated by dots in each map.

13

ARRIVING AT A THIRD-QUARTER MOON

All of the transfers studied so far have arrived at the Moon at the same time, when the Moon is at its first quarter. Yet spacecraft missions may need to arrive at the Moon at any time of the month. As a second step in this survey, lunar transfers are studied that arrive at the Moon on August 3rd, 2010 at 04:38:29 ET: a time when the Moon has reached its third quarter. Figure 12 shows two example transfers that arrive at the third-quarter Moon, where the trajectory on the left is a direct lunar transfer and the trajectory on the right is a low-energy transfer. Neither transfer requires any extra Earth phasing orbits or lunar flybys. One notices that the low-energy transfer extends away from the Sun rather than toward it as seen in Figures 3 and 7. Otherwise the transfers appear very similar to those studied previously. The symmetry observed here is expected according to the nearly symmetrical dynamics in the Sun-Earth system.[9] The Sun-Earth L_1 and L_2 points are located nearly the same distance from the Earth, and three-body libration orbits about those Lagrange points behave in a very similar fashion.[4]
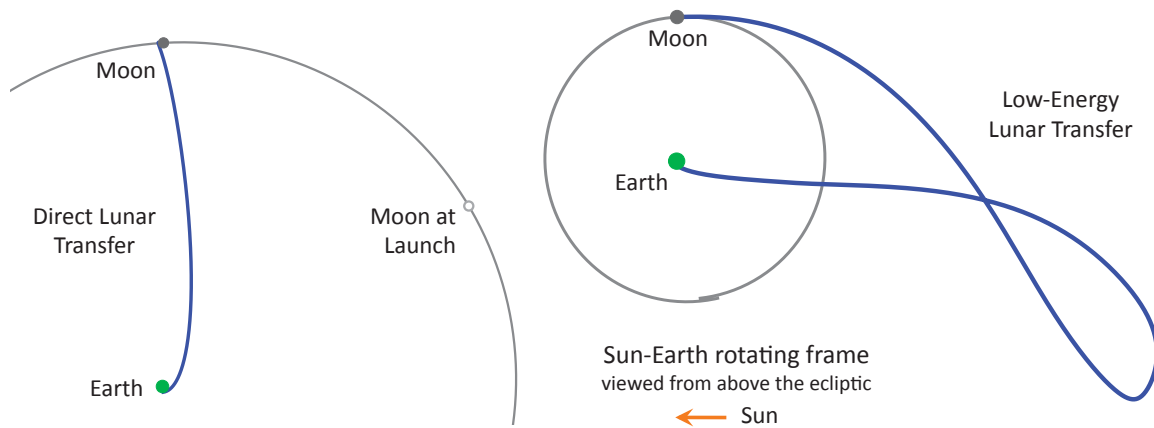

Figure 12   Two example lunar transfers that arrive at a third-quarter Moon. The transfers are simple, direct (left) and low-energy (right) lunar transfers with no Earth phasing orbits nor lunar flybys. The transfers are viewed from above in the Sun-Earth rotating frame of reference.

One may construct state space maps for transfers to a third-quarter Moon in the same way that maps have been constructed previously to a first-quarter Moon. Figures 13 and 14 plot state space maps for transfers to target orbits with Ω-values of $0° - 80°$ and $180° - 260°$, respectively. These ranges of Ω-values track the interesting features as the orbit plane changes; the maps of the Ω-values between those plotted in the figures vary little across the range. An Ω-value of $126.9°$ (also $306.9°$) is as close to orthogonal to the Earth-Moon axis as a polar orbit can be at this time. Transfers within about $60°$ of this angle are all very similar, though the cost of those transfers rises as the orbital plane moves away from this optimal Ω-value. When one compares the maps shown in Figures 13 and 14 to those constructed earlier in Figures 8 and 9, one sees that the maps are very similar with a $180°$ plane change. This makes sense, of course, since the transfers are arriving at the Moon when it is $180°$ further along in its orbit, while the inertial coordinate axes that define Ω and ω have not changed.

Figure 15 shows the same two plots as shown in Figure 11 for these third-quarter lunar arrival transfers. The maps show the LOI ΔV cost and transfer duration for simple lunar transfers that target

different lunar orbits. As before, if there are multiple lunar transfers that may be used to arrive at the same lunar orbit, then the maps present the parameters for the transfer with the least LOI ΔV. The maps illustrate that the same trends exist to third-quarter lunar arrivals as do to first-quarter lunar arrivals, but with a $180°$ shift in Ω.
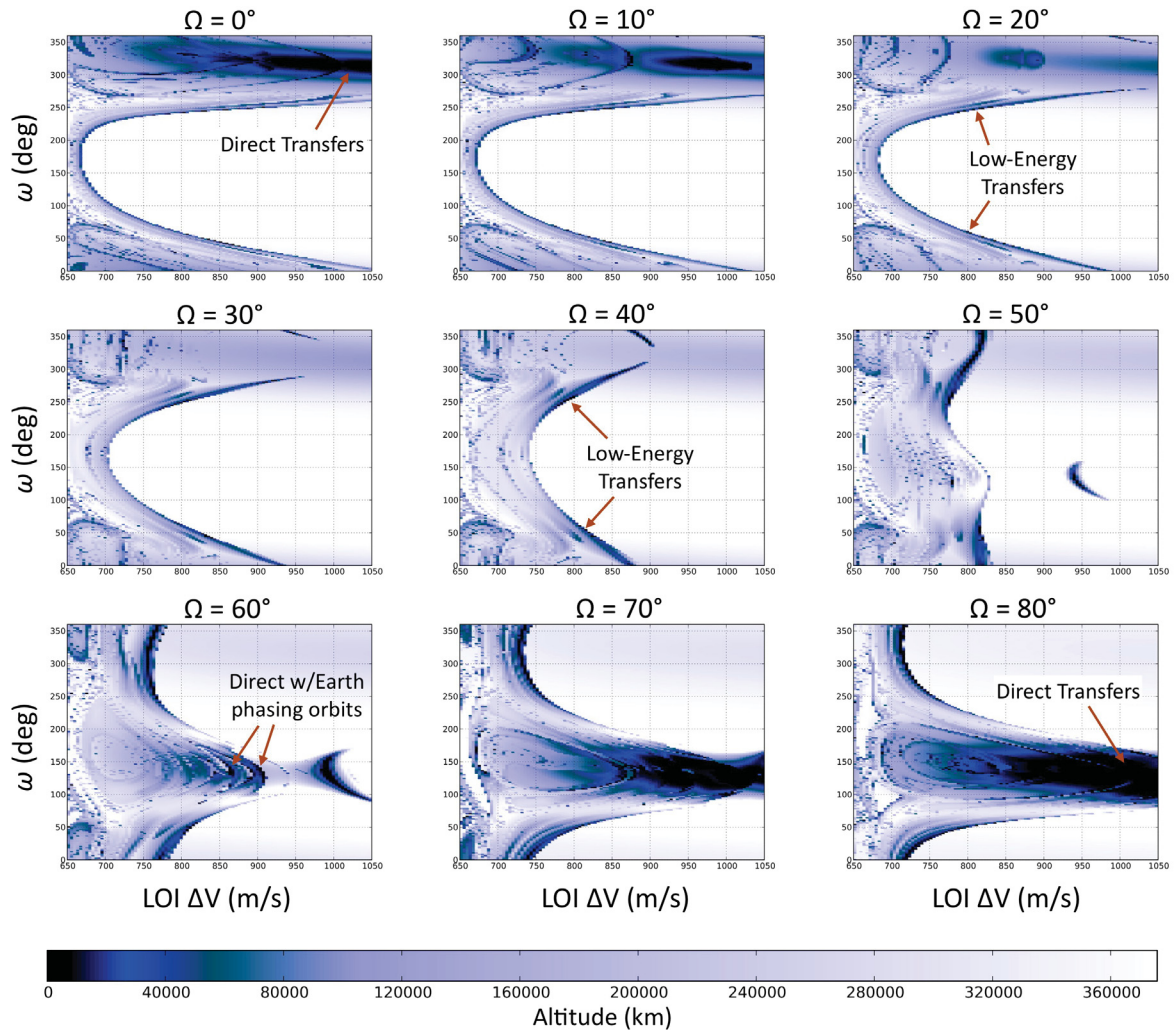

Figure 13   Nine surveys of missions that arrive at the third-quarter Moon, where the target orbit's Ω varies from $0° - 80°$. The points are again colored according to how close they approach to the Earth when propagated backward in time, using the same color scheme applied in previous maps.

ARRIVING AT A FULL MOON

Trajectories have been studied that arrive at the Moon when the Sun-Earth-Moon angle is near $90°$; this section briefly considers trajectories that arrive at a full Moon, when the Sun-Earth-Moon angle is approximately $180°$. Lunar transfers that arrive at a new Moon have much the same characteristics as those that arrive at a full Moon, but with a familiar $180°$ shift in Ω; they will not be shown here for brevity.
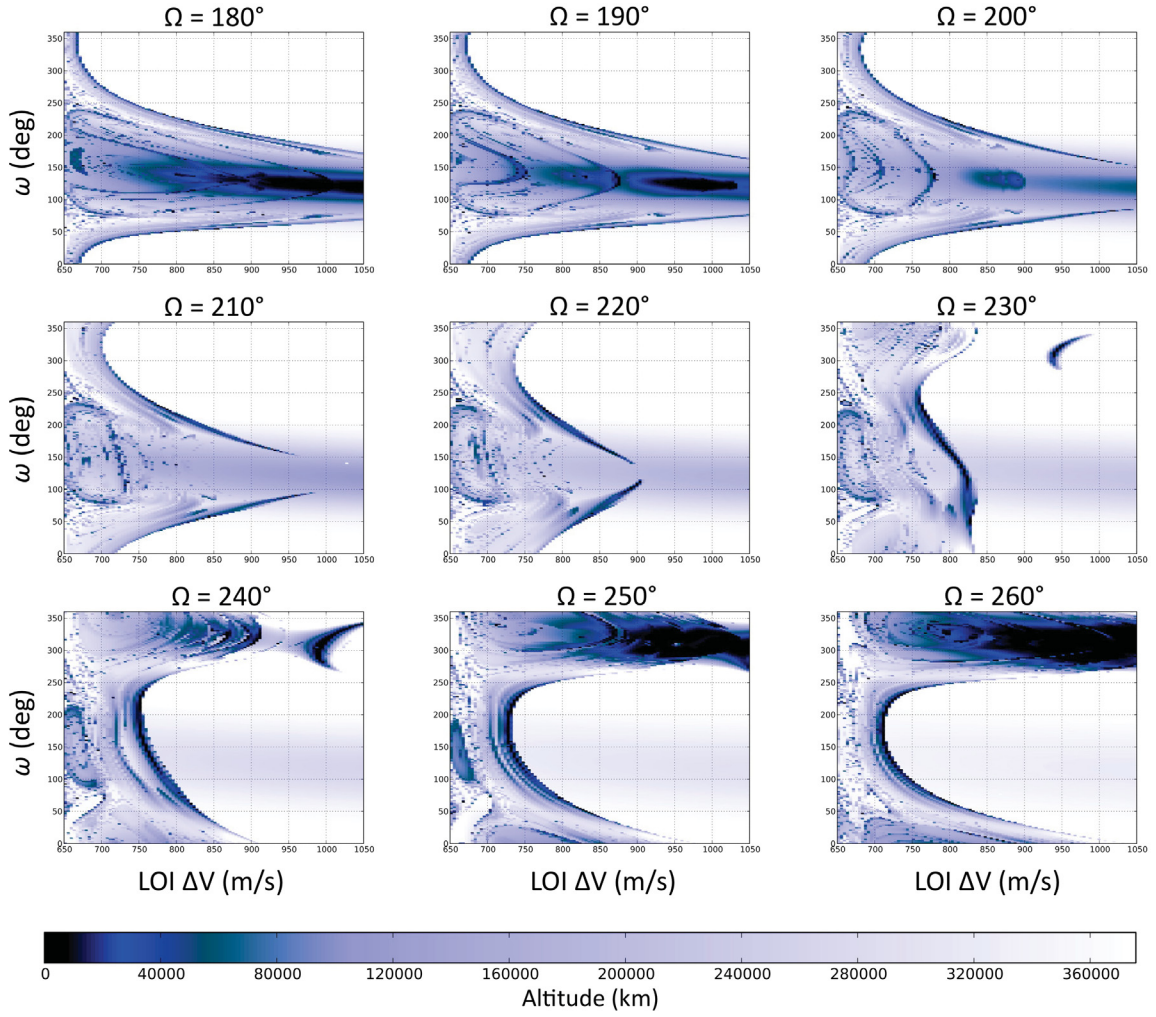
Figure 14   Nine surveys of missions that arrive at the third-quarter Moon, where the target orbit's Ω varies from 180° − 260°.

Figures 16 and 17 present state space maps for trajectories that arrive at the full Moon in polar orbits with Ω-values in the ranges 90° − 170° and 270° − 350°, respectively. The maps not shown vary only gradually between these maps. One observes that direct lunar transfers arrive at the full Moon with low-ΔV insertions at Ω-values approximately 90° apart from those that arrive at the first-quarter and third-quarter Moons. This demonstrates additional evidence that the minimum orbit insertion ΔV requirements for direct lunar transfers occurs when the orbit's plane is nearly orthogonal to the Earth-Moon line.

The low-energy lunar transfers' locations in the state space maps evolve somewhat differently as Ω varies compared with their evolutions in the state space maps for transfers to first- and third-quarter Moons. Low-energy transfers still arrive at the Moon for any Ω-value, but the range of ω-values that may be used are bifurcated along the range of Ω-values. Many of the low-energy transfers that require the least LOI ΔV arrive at the full Moon at ω-values near 75° and 255°. These transfers fly further out of the plane of the Moon's orbit than others; those transfers that remain closer to the Moon's orbital plane require more ΔV and target ω-values near 165° and 345°.
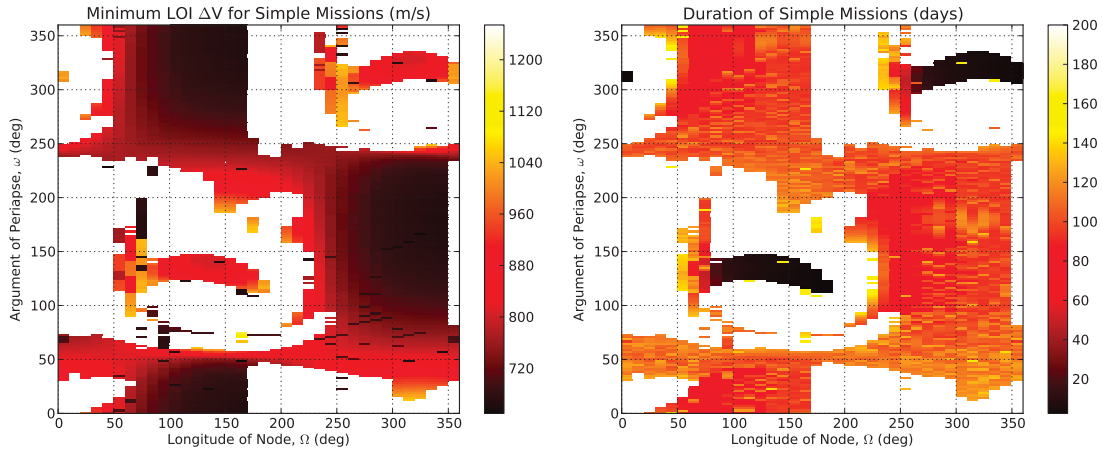
16

Figure 15 **The combinations of Ω and ω that yield simple lunar transfers, i.e., those without low Earth or lunar periapse passages. If multiple transfers exist for the same combination, then the one with the least LOI ΔV is shown. All of these transfers arrive at a third-quarter Moon.**

MONTHLY TRENDS

Most characteristics of ballistic, two-burn lunar transfers, such as those presented in this paper, repeat from one month to the next. The Moon's orbital plane is nearly coplanar to the Earth's, and the orbits of the bodies involved are nearly circular. However, since these conditions are not perfectly met, the characteristics of these lunar transfers do vary from one month to the next. The Moon's orbital plane is approximately $5.1°$ tilted with respect to the ecliptic, and the Moon's spin axis has an obliquity of approximately $1.5°$ relative to the ecliptic. One may therefore assume that the characteristics observed in the state space maps presented here vary by several degrees in their ω-values. Furthermore, the inclination of the trans-lunar departure state for a given type of lunar transfer may vary by many degrees from one month to the next, due to the reasons mentioned, but especially on account of the obliquity of the Earth's spin axis.

It has been found that most types of *simple* lunar transfers appear in any given month. The more complex lunar transfers, such as those with multiple lunar flybys, vary much more on a monthly basis and may not even appear at all in a given month. It is very interesting to observe that simple low-energy transfers may be used to target lunar orbits with any Ω-value, though their ΔV requirements vary. It is also interesting that simple, short-duration, direct lunar transfers have the least LOI ΔV for any arrival time or month when targeting lunar orbits that are close to orthogonal to the Earth-Moon line.

PRACTICAL CONSIDERATIONS

The surveys presented here study trajectories that are entirely ballistic - they do not contain any trajectory correction maneuvers or targeting maneuvers of any sort. When propagated backward in time from the Moon, if a trajectory arrives at the Earth without impacting the Moon then it is considered a viable Earth-Moon transfer. However, the trajectory may have arrived at the Earth with an inclination that is unsuitable for a mission that launches from a particular launch site. Ideally, a mission would start in a low-Earth parking orbit with an inclination very close to that of the latitude
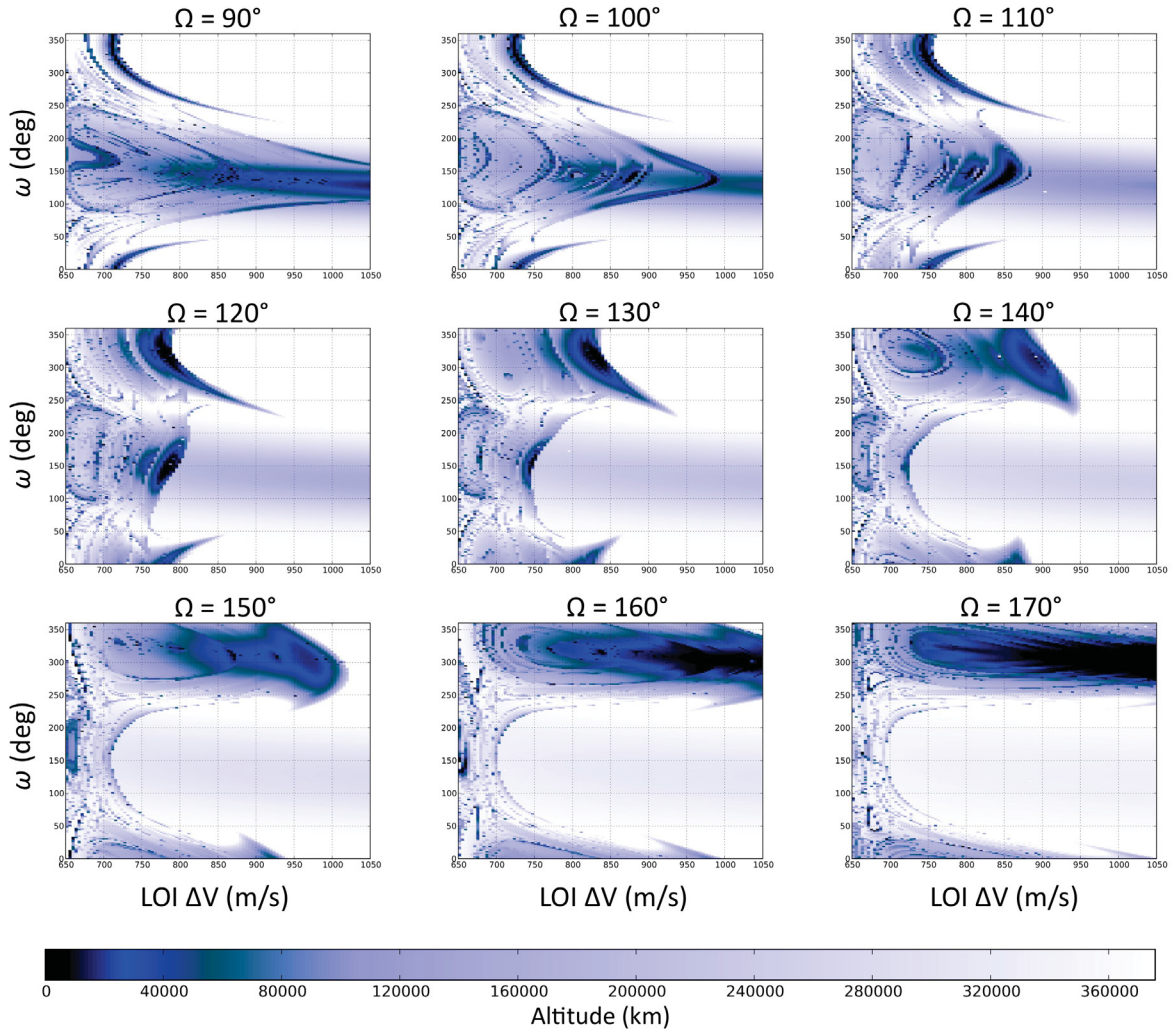
Figure 16 Nine surveys of missions that arrive at a full Moon, where the target orbit's Ω varies from $90° - 170°$.

of the launch site, e.g., near 28.5° for missions that launch from Cape Canaveral. It is undesirable to perform a large plane change during launch and trans-lunar injection. Previous work[8] has found that one can add $1 - 3$ small trajectory correction maneuvers to depart the Earth from a particular LEO parking orbit and transfer onto a desirable *low-energy* transfer to the Moon; and doing so requires only about 1 m/s per degree of inclination change. This works for low-energy transfers particularly well since low-energy transfers travel far from the Earth and spend many weeks doing so. This method does not work well for direct lunar transfers, which require far more ΔV to change planes.

Trajectory correction maneuvers may also be implemented to establish a launch period for a low-energy transfer to the Moon. Missions that implement direct lunar transfers may establish a launch period using Earth phasing orbits, making those sorts of transfers more desirable in the surveys presented here.
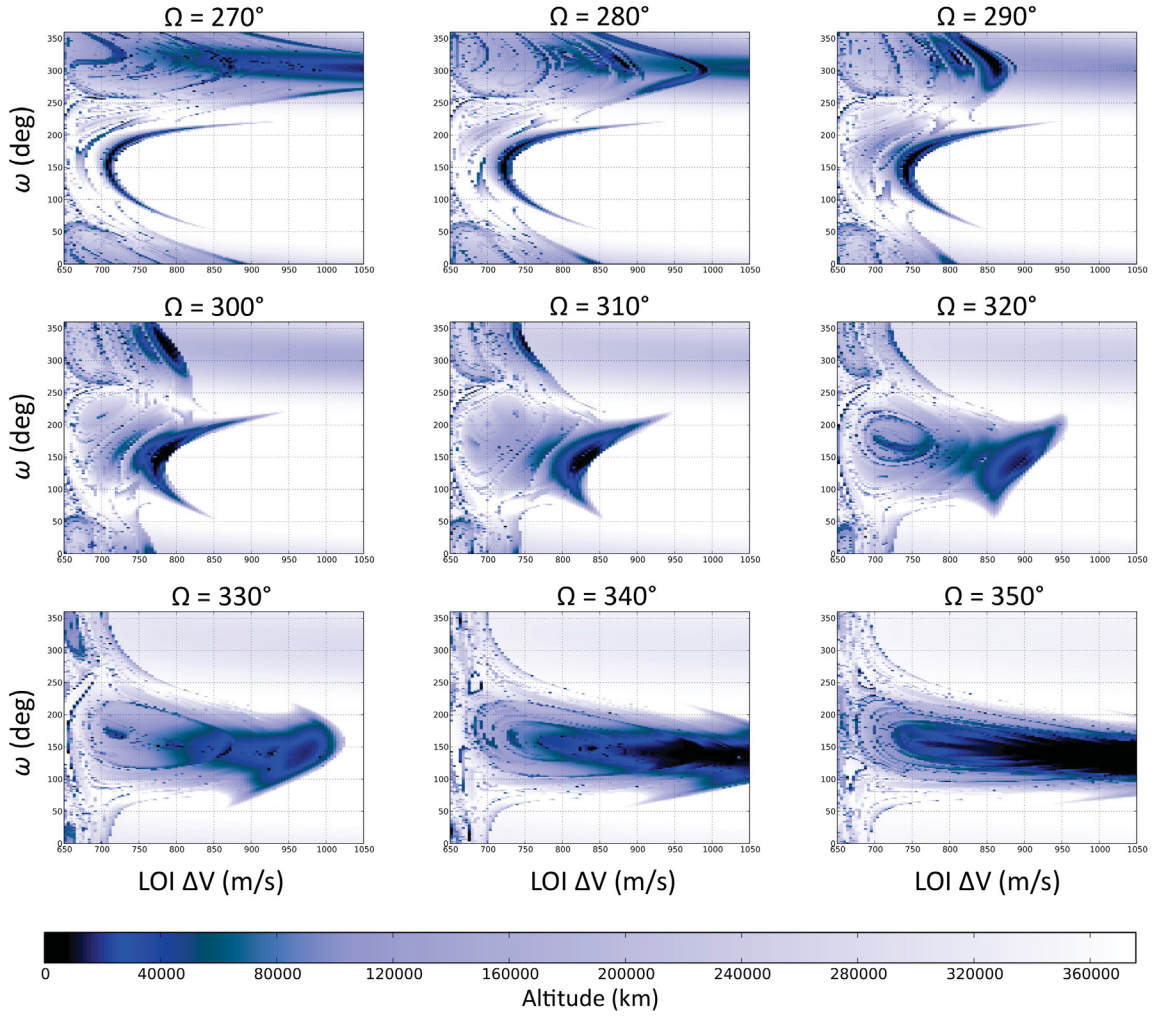
Figure 17 Nine surveys of missions that arrive at a full Moon, where the target orbit's Ω varies from 270° − 350°.

CONCLUSIONS

The surveys presented in this paper characterize two-burn lunar transfers that arrive at the Moon, targeting 100-km polar orbits with any orientation. Transfers are studied that arrive at an example first-quarter Moon, an example full Moon, and an example third-quarter Moon. Many types of transfers are observed, including low-energy transfers, short-duration direct transfers, and variations that involve any number of lunar flybys and Earth phasing orbits, permitted that they do not involve any deterministic maneuvers. The only two burns considered are the trans-lunar injection maneuver and orbit insertion maneuver.

It has been found that lunar transfers consistently require trans-lunar injection C_3 values on the order of -2.0 km^2/s^2 for direct transfers and -0.7 km^2/s^2 for low-energy transfers. Simple transfers require 2–12 days for direct transfers and 70–120 days for low-energy transfers. The low-energy transfers that require the least LOI ΔV require 660–780 m/s, depending on the target orbit and the arrival time; direct lunar transfers require at least 140 m/s more ΔV. Practical simple direct transfers only exist that target a lunar orbit that is within 60° of being orthogonal to the Earth-Moon
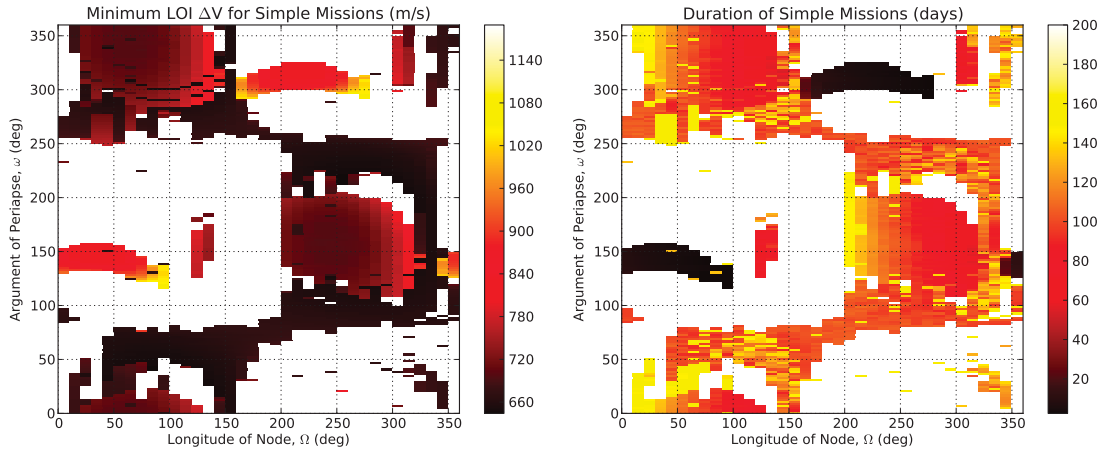
Figure 18   The combinations of Ω and ω that yield simple lunar transfers, i.e., those without low Earth or lunar periapse passages. If multiple transfers exist for the same combination, then the one with the least LOI $\triangle$V is shown. All of these transfers arrive at a full Moon.

line, though the $\triangle$V cost rises significantly when the orbit is beyond $30°$ of orthogonal. Low-energy transfers can target polar orbits with any argument of periapse, ω, or with any longitude of ascending node, Ω; though targeting one such parameter restricts the other for a particular arrival date as illustrated in the state space maps presented here.

ACKNOWLEDGMENTS

REFERENCES

[1] R. B. Roncoli and K. K. Fujii, "Mission Design Overview for the Gravity Recovery and Interior Laboratory (GRAIL) Mission," *AIAA Guidance, Navigation, and Control Conference*, No. AIAA 2010-8383, Toronto, Ontario, Canada, AIAA, August 2–5, 2010.

[2] M. J. Chung, S. J. Hatch, J. A. Kangas, S. M. Long, R. B. Roncoli, and T. H. Sweetser, "Trans-Lunar Cruise Trajectory Design of GRAIL (Gravity Recovery and Interior Laboratory) Mission," *AIAA Guidance, Navigation, and Control Conference*, No. AIAA 2010-8384, Toronto, Ontario, Canada, AIAA, August 2–5, 2010.

[3] S. J. Hatch, R. B. Roncoli, and T. H. Sweetser, "Trans-Lunar Cruise Trajectory Design of GRAIL (Gravity Recovery and Interior Laboratory) Mission," *AIAA Guidance, Navigation, and Control Conference*, No. AIAA 2010-8385, Toronto, Ontario, Canada, AIAA, August 2–5, 2010.

[4] J. S. Parker, *Low-Energy Ballistic Lunar Transfers*. PhD thesis, University of Colorado, Boulder, Colorado, 2007.

[5] J. N. Goswami and M. Annadurai, "Chandrayaan-1: India's First Planetary Science Mission to the Moon," *Current Science; Special Section: Chandrayaan-1*, Vol. 96, 25 February 2009, pp. 486–491.

[6] Flight Planning Branch, Flight Crew Support Division, "Apollo 11 Flight Plan," Tech. Rep. AS-506 / CSM-107 / LM-5, NASA, Manned Spacecraft Center, Houston, Texas, July 1, 1969.

[7] D. A. Vallado, *Fundamentals of Astrodynamics and Applications, Second Edition*. Microcosm Press, El Segundo, CA, and Kluwer Academic Publishers, Dordrecht/Boston/London, 2001.

[8] J. S. Parker, "Targeting Low-Energy Ballistic Lunar Transfers," *American Astronautical Society, George H. Born Special Symposium*, Boulder, Colorado, AAS, May 13–14, 2010.

[9] V. Szebehely, *Theory of Orbits: The Restricted Problem of Three Bodies*. New York: Academic Press, 1967.